



DCMT101

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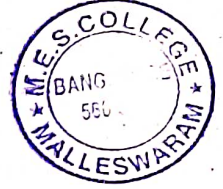
I Semester B.Sc. Degree Examination, April - 2023

MATHEMATICS

Algebra - I and Calculus - I

Paper : I

(NEP Core Scheme)



Time : 2½ Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all questions.

I. Answer any SIX questions.

(6×2=12)

1. Find the value of k for which the system of equations $2x - y + 2z = 0$; $3x + y - z = 0$; $kx - 2y + z = 0$ has non trivial solutions.

2. Find the Eigen values of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

3. Find the n^{th} derivative of $e^{2x} \sin 3x$.

4. If $u = x^2 + y^2 - 3xy$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

5. Find the value of k such that the function $f(x) = \begin{cases} kx^2 & \text{if } x > 2 \\ 8 & \text{if } x \leq 2 \end{cases}$ is continuous at $x=2$.

6. State Euler's theorem for the homogeneous function.

7. Verify Rolle's theorem for the function $f(x) = 8x - x^2$ in $[2,6]$.

8. Evaluate : $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$, using L'Hospital rule.

II. Answer any THREE of the following.

(3×4=12)

9. Find the rank of matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing to echelon form.

[P.T.O.]



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10. Solve completely the system of equations $x + 2y - 4z = 0$; $3x - y + 2z = 0$ and $5x - 3y + z = 0$.

11. Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

12. State and prove Caley - Hamilton theorem.

13. Using Caley - Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

III. Answer any THREE questions.

(3×4=12)

14. Evaluate $\lim_{x \rightarrow 0} \frac{e^{1/x}}{1 + e^{1/x}}$.

15. Examine the differentiability of the function $f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases}$ at $x=1$.

16. Prove that a function which is continuous on a closed interval is bounded.

17. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$.

18. If $y = \sin^{-1} x$ prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

IV. Answer any THREE questions.

(3×4=12)

19. State and prove intermediate value theorem.

20. State and prove Rolle's theorem.

21. Find the Taylor expansion of $f(x) = \cos x$ about the point $x = \frac{\pi}{2}$ upto 4th degree terms.

22. Expand $\log(1+x)$ upto the term containing x^4 using Maclaurin's series.

23. Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$ using L'Hospital rule.

V. Answer any THREE questions.

(3×4=12)

24. If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \cdot \frac{\partial^2 z}{\partial y^2}$.

25. If $z = x^2 + y^2$ where $x = e^t \cos t$, $y = e^t \sin t$ find $\frac{dz}{dt}$.

26. Verify Euler's theorem for the function $u = ax^2 + 2hxy + by^2$.

27. Expand $e^x \sin y$ in powers of x and y upto second degree terms.

28. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.